Time dilation.

A moving clock runs slower than a clock at rest!

Suppose that there is a a clock moving with uniform velocity in the moving reference frame and a stationary observer in a non-moving reference frame. Assume the velocity of light, c = 1, and the velocity v of the moving reference frame is measured in terms of c, the speed of light. Here is the pertinent space-time diagram:



Spacetime graph: affect on Time and Space axes when inertial frame velocity = 0.50 times the speed of light,

In the moving frame of reference, by the Lorentz Transform we have $x = \frac{(x'+vt')}{\sqrt{1-v^2}}$ and $t = \frac{(t'+vx')}{\sqrt{1-v^2}}$.

Suppose t' = 1 on the clock of a moving observer located at x' = 0 who is moving at velocity v relative to an observer in a stationary reference frame.

Then,
$$t = \frac{(t'+vx')}{\sqrt{1-v^2}} = \frac{(1+v\cdot 0)}{\sqrt{1-v^2}} = \frac{1}{\sqrt{1-v^2}}$$
. Clearly $t = \frac{t'}{\sqrt{1-v^2}} > t'$.

 \therefore time is dilated for stationary clocks by a factor of $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.$